Novosad Z.H.

CHAOTIC DYNAMIC SYSTEMS OF SHIFT OPERATORS AND APPLICATIONS IN ECONOMICS

In this paper we consider chaotic properties of weighted shifts on (non-separable) Hilbert space. We ivestigate some conditions under which the operators are Li-Yorke chaos. We examine various structural of the operators that contribute to their chaotic behavior, providing theoretical results that highlight the interplay between the weights and the underlying space. Also, we construct chaotic dynamic system for modeling the security price.

Key words and phrases: dynamic system, chaotic operator, hypercyclic operator, Hilbert space.

Lviv University of Trade and Economics, 10, Tuhan-Baranovsky Str., Lviv 79005, Ukraine
e-mail: *zoriana.maths@gmail.com, zoryana@lute.lviv.ua*

Introduction

Chaos theory has emerged as a vital area in the study of dynamical systems, focusing on the unpredictable and complex behavior exhibited by certain deterministic systems. A chaotic dynamical system is one that demonstrates sensitive dependence on initial conditions, topological transitivity, and a dense set of periodic points. These properties together produce dynamics that are seemingly random and yet governed by deterministic rules.

In operator theory, the concept of chaos is closely tied to chaotic operators, which extend the notion of chaos from classical systems to functional spaces. An operator is termed chaotic if it is hypercyclic and has a dense set of periodic points. Hypercyclicity, a foundational concept in this area, refers to the existence of a vector in a function space whose orbit under repeated applications of the operator is dense in that space. This property, first studied in depth by Birkhoff and MacLane, illustrates how linear operators can exhibit behavior analogous to classical chaotic systems. The two of examples of classical chaotic systems

The Lorenz System.

УДК 517
2010 *Mathematics Subject Classification:* 47A16, 46E10, 46E50.
The research was carried out as part of the scientific project "Modeling and Application of Chaotic Dynamical Systems for Optimizing Algorithms of Artificial Intelligence, Machine Learning, and Web Technologies" (state registration number 0124U004447).

Described by Edward Lorenz in 1963, this system of three coupled nonlinear differential equations is a classic example of chaos. The Lorenz attractor exhibits a distinctive butterfly-shaped fractal structure.

The Logistic Map.

An iterative map $x_{n+1} = rx_n(1 - x_n)$ that demonstrates chaos for certain values of the parameter r. Despite its simplicity, it shows complex dynamics, including bifurcations and chaotic behavior.

As is often the case in linear dynamics, the concepts mentioned above have been extensively studied by researchers within the framework of a particular class of operators known as weighted shifts. This is largely due to their flexibility in constructing examples in linear dynamics, operator theory, and its various applications. Over recent decades, numerous dynamical properties of such operators have been thoroughly examined and described [1], [2], [3] [4], [10], occasionally even before these properties were fully understood in broader contexts. Specifically, in [5] and [6], the authors advanced this field by offering detailed analyses of chaotic properties.

We consider chaotic properties such as

$$Topologacal \quad transitivity \quad \Rightarrow Hypercyclicity \quad \Rightarrow \quad Li-Yorke \quad chaos$$

$$Frequent \quad hypercyclicity \quad \Rightarrow Hypercyclicity \quad \Leftarrow \quad Chaos$$

In Section 1 we consider chaotic properties of weighted shifts on (non-separable) Hilbert space and ivestigate some conditions under which the operators are Li-Yorke chaos. In Section 2 we construct a dynamic system based on these operators to model the price behavior of financial securities. This application demonstrates the practical relevance of chaotic dynamics in economics, where security prices often exhibit irregular, unpredictable fluctuations. This study bridges the gap between abstract operator theory and applied financial modeling, opening new avenues for both mathematical and economic research.

1 Backward Shifts for Banach Spaces

Let X be a metric space and T be a continuous mapping $T\colon X \to X$. T is called *topologically transitive* if, for any pair U, V, ($U \neq \varnothing$, $V \neq \varnothing$) of open subsets of X, there exists some integer $k \geq 0$ such that $T^k(U) \cap V \neq \varnothing$.

A sequence of closed subspaces (X_n) of a Banach space X is called a *Schauder decomposition* of X if every element $x \in X$ can be expressed uniquely as a sum of elements from these subspaces

$$x = \sum_{k=0}^{\infty} x_k, \quad x_k \in X_k, \tag{1}$$

and the series (1) converges in X. A Schauder decomposition (X_n) is *unconditional* if (1) converges unconditionally.

In [2], it was pointed out that a criterion for topological transitivity, analogous to that for hypercyclic operators [1, 7], can be formulated.

Theorem 1. (see for the proof e.g. [8]). *Let T be a bounded linear operator on a Banach space X (not necessarily separable). Suppose that there exists a strictly increasing sequence (n_k), $\lim_{k\mapsto\infty} n_k = \infty$ of positive integers for which there are the following:*

(i) A dense subset $Z_0 \subset X$ such that $T^{n_k}(x) \to 0$ for every $x \in Z_0$ as $k \to \infty$.

(ii) A dense subset $Y_0 \subset X$ and a sequence of mappings (not necessary linear and not necessary continuous) $S_k \colon Y_0 \to X$ such that $S_k(y) \to 0$ for every $y \in Y_0$ and $T^{n_k} \circ S_k(y) \to y$ for every $y \in Y_0$ as $k \to \infty$.

Then, T is topologically transitive.

We consider an infinite-dimensional (may be non-separable) Banach space X which admits an unconditional Schauder decomposition to Banach spaces X_k, $k = 0, 1, \ldots$. Let $(J_k)_{k=1}^{\infty}$ be a sequence of injective maps $J_k \colon X_{k+1} \to X_k$ with dense ranges and $\|J_k\| = 1$. We have the following shifts of spaces X_k under maps J_k:

$$0 \longleftarrow X_0 \overset{J_1}{\longleftarrow} X_1 \overset{J_2}{\longleftarrow} \cdots \overset{J_n}{\longleftarrow} X_n \cdots .$$

Let us construct a weighted backward shift operator *(associated with a Schauder decomposition (X_n) of X)* by

$$T(x) = \sum_{k=1}^{\infty} \omega_k J_k(x_k), \tag{2}$$

$$T \colon (x_0, x_1, \ldots, x_n, \ldots) \mapsto (\omega_1 J_1(x_1), \omega_2 J_2(x_2), \ldots, \omega_n J_n(x_n), \ldots),$$

where (ω_k) is a sequence of positive numbers with $\sup_k \omega_k < \infty$.

Theorem 2. *([9]) Let X be a Banach space that can be represented as an unconditional Schauder decomposition into Banach spaces X_k, $k = 0, 1, \ldots$ and T a weighted backward shift, defined as in (2). Assume that the following conditions are satisfied*

(i) The weight constants ω_k are such that

$$\limsup_{n\to\infty} \prod_{k=1}^{n} \omega_k = \infty.$$

(ii) There is a dense subspace $E_0 \subset (J_1) \subset X_0$ such that for every $x \in E_0$ the set

$$\{J_n^{-1} \circ \cdots \circ J_1^{-1}(x), \quad n \in \mathbb{N}\}$$

is bounded in X.

Then the operator T defined by (2) is topologically transitive.

2 Chaotic Properties of Weighted Shifts

Let $(H_n)_{n=0}^{\infty}$ be a sequence of Hilbert spaces. In this paper, we assume that each H_n is nontrivial, meaning $H_n \neq \{0\}$ and may not necessarily be separable.

Assume that for all n and m, the spaces H_n and H_m are isomorphic. We define $\ell_2(H_n) = \ell_2((H_n)_{n=0}^{\infty})$ as the Hilbert space consisting of elements $x = (x_0, x_1, \ldots, x_n, \ldots)$, $x_k \in H_k$ endowed with norm $\|x\| = \left(\sum_{i=0}^{\infty} \|x_i\|^2 \right)^{\frac{1}{2}}$.

Let (ω_n) be a sequence of positive numbers, referred to as weights. Additionally, let us fix a sequence of isomorphisms $J_m : H_m \to H_{m-1}$, $\|J_m\| = 1$, $m \in \mathbb{N}$. An operator

$$T\colon \ell_2(H_n) \to \ell_2(H_n)$$

will be called a *backward weighted shift (with respect to the family (J_m)) with weight sequence (ω_n)* if it is of the form

$$T(x) = (\omega_1 J_1(x_1), \omega_2 J_2(x_2), \ldots, \omega_m J_m(x_m), \ldots).$$

We will need the next corollary which is proved in [12].

Corollary 1. *([12]) Let $(H_n)_{n=0}^{\infty}$ be a sequence of Hilbert spaces and $T\colon \ell_2(H_n) \to \ell_2(H_n)$ be a backward weighed shift with respect to (J_m) and with positive weight sequence (ω_n). Let us suppose that*

$$\sup_{m \in \mathbb{Z}_+} \prod_{n=0}^{m} \|J_n^{-1}\| < \infty. \tag{3}$$

Then the following are equivalent:

(i) T is topologically transitive.

(ii) There exists a non-trivial T-invariant (separable) closed subspace $\mathcal{Y} \subset \ell_2(H_n)$ on which the restriction of T to $\mathcal{Y}$, $T : \mathcal{Y} \to \mathcal{Y}$, is hypercyclic.

(iii) The restriction $T : \mathcal{Y} \to \mathcal{Y}$ to any T-invariant (separable) closed subspace $\mathcal{Y} \subset \ell_2(H_n)$ which contains non-zero vectors of the form $(0, \ldots, 0, x_n, 0, \ldots)$, $x_n \in H_n$ for every $n \in \mathbb{Z}_+$, is hypercyclic.

(iv) $\limsup_{n \to \infty} \prod_{k=1}^{n} \omega_k = \infty.$

Let $T : X \mapsto X$ be a bounded linear operator acting on topological space X

Definition 1. *The operator T is*

- *Li-Yorke chaotic if there is uncountable set $U \subset X$, called scrambled set, such that for each $x, y \in U$, $x \neq y$, $\lim_{n \mapsto \infty} \|T^n(x) - T^n(y)\| = 0$*

- *Hypercyclic if there is a vector* $x \in X$ *for which the orbit under* T, $\text{Orb}(T, x) = \{x, Tx, T^2x, \ldots\}$ *is dense in* X*. Every such vector* x *is called a hypercyclic vector of* T*.*

- *Frequently hypercyclic if* T *admits a frequently hypercyclic vector* $x \in X$ *such that for each non-empty open subset* U *of* X, $\lim\limits_{N \mapsto \infty} \frac{\{1 \le n \le N : T^n(x) \in U\}}{N} > 0$.

1. Li-Yorke Chaos.

An operator T is Li-Yorke chaotic if there exists an uncountable set $S \subset \ell_2(H_n)$ such that for any $x, y \in S$, $x \neq y$, (points are not asymptotic), $\limsup\limits_{n \to \infty} \|T^n(x-y)\| > 0$ (points are proximal). For the weighted shift T, we note that the weights $(\omega)_n$ control the growth/decay of iterates. If $\prod\limits_{i=1}^{n} \omega_k \to 0$ or diverges, then the distance between certain points can oscillate, fulfilling the conditions for Li-Yorke chaos. The isometries J_m preserve structure, allowing T to meet the chaotic requirements under appropriate ω_n. So, we can state the next theorem.

Theorem 3. *Let* $(H_n)_{n=0}^{\infty}$ *be a sequence of Hilbert spaces. An operator*

$$T \colon \ell_2(H_n) \to \ell_2(H_n)$$

$$T(x) = (\omega_1 J_1(x_1), \omega_2 J_2(x_2), \ldots, \omega_m J_m(x_m), \ldots)$$

is Li-Yorke chaotic

Proof. For any $x = (x_1, x_2, \ldots) \in \ell_2(H_n)$ the action of T is given by

$$T(x) = (\omega_1 J_1(x_1), \omega_2 J_2(x_2), \ldots, \omega_m J_m(x_m), \ldots),$$

then the n-th iterate $T^n(x)$ is

$$T^n(x) = (\omega_n J_n(x_n), \omega_{n+1} J_{n+1}(x_{n+1}), \ldots)).$$

with norm

$$\|T^n(x)\| = \left(\sum_{k=n}^{\infty} \omega_k^2 \|J_k(x_k)\|^2 \right)^{1/2}.$$

For vectors $x, y \in \ell_2(H_n)$, consider $z = x - y$. The norm satisfies

$$\|T^n(z)\| = \left(\sum_{k=n}^{\infty} \omega_k^2 \|J_k(z_k)\|^2 \right)^{1/2}.$$

The weights (ω_k) and the dynamics of J_k (3) can ensure such behavior that $\|T^n(z)\|$ alternates between being arbitrarily small and bounded away from zero, depending on the decay or growth of the weights (ω_k)

We can constract uncountable set $S \subset \ell_2(H_n)$ with the properties by leveraging the oscillatory behavior of T iterates. Choose S as a subset of $\ell_2(H_n)$ with coordinates that exhibit chaotic pairing behavior under T.

This typically involves specific properties of (ω_n) and the shifts J_n to create a scrambled set where the required, using the colorally 1 we will have that if $\limsup_{n\to\infty} \prod_{k=1}^{n} \omega_k = \infty$ the norm of $T^n(x)$ for certain vectors x grows arbitrarily large for some iterations n This ensures that the separation condition for a scrambled set $\limsup_{n\mapsto\infty} \|T^n(x-y)\| > 0$ is satisfied.

So,

$$\limsup_{n\mapsto\infty} \|T^n(x-y)\| > 0,$$

and

$$\liminf_{n\mapsto\infty} \|T^n(x-y)\| = 0$$

conditions are satisfied. Thus, T is Li–Yorke chaotic. □

2. Chaos.

An operator T is chaotic if there exists a dense set of periodic points.

It is hypercyclic (there exists $x \in \ell_2(H_n)$ such that $\{T^n(x) : n \geq 0\}$ is dense in $\ell_2(H_n)$. For proving of we need find x whose iterates under T approximate any $x \in \ell_2(H_n)$ and show density of periodic points by solving $T^n(x) = x$ for suitable $x \in X$.

For $x \in \ell_2(H_n)$ we define periodic sequences x_k such that only finitely many coordinates x_k are nonzero. These sequences are in $\ell_2(H_n)$ and are clearly periodic under T. The set of such periodic points is dense in $\ell_2(H_n)$ because any vector in $\ell_2(H_n)$ can be approximated arbitrarily closely by a vector with finitely many nonzero coordinates.

So, the operator T satisfies both conditions for Li–Yorke chaos that an uncountable scrambled set S exists due to the oscillatory nature of norms under T^n. The set of periodic points is dense in $\ell_2(H_n)$.

3. Frequent Hypercyclicity.

An operator T is frequently hypercyclic if there exists $x \in \ell_2(H_n)$ such that for every open set $U \in \ell_2(H_n)$, the set $\{n \geq 0 : T^n(x) \in U\}$ has positive lower density.

For proving this statement we need to note that weighted shifts, the growth of (ω_n) ensures the existence of frequently hypercyclic vectors x.

Construct x with nonzero components in H_n that balance the effect of weights.

3 Dynamic systems for modeling of security price.

In this section, we will consider two examples of dynamic systems in the securities market. Let us begin with the definition of a discrete dynamic system.

Definition 2. *A (discrete) dynamical system is a pair (X, T) consisting of a metric space X and a continuous map $T : X \to X$.*

Sometimes we will simply call $T : X \to X$ a dynamical system.

Example 1. *(***Effectiveness of securities***).*

We assume to be given by the value C_n (security price) at discrete times $n = 1, 2, \dots$. In a simple model the security price at time $n+1$ will only depend on the security price at time n. The effectiveness of securities is then described by a law

$$C_{n+1} = T(C_n), \quad n = 1, 2, \dots$$

where T is suitable map. It follows that

$$C_n = (T \circ \cdots \circ T)(C_1) \quad n = 1, 2, \dots$$

with n applicants of the map. Thus the behavior of the security price is completely determined by the initial price C_1 and the map T.

Now we consider the proportional change of security price.

Let C_n be purchase price of the securities, C_{n+1} be sale price of the securities and period of time $n = 1, 2, \dots$.

We suppose that the value C_n of a price changes proportionally to its actual value, that is follows the law

$$\frac{C_{n+1} - C_n}{C_n} = \gamma, \quad n \geq 1$$

where γ is effectiveness of security, $\gamma > -1$.

One may write this equivalently as

$$C_{n+1} = (1 + \gamma) C_n$$

so that the corresponding dynamical system is given by

$$T : \mathbb{R}_+ \mapsto \mathbb{R}_+, \quad Tx = (1 + \gamma)x.$$

The orbit of $x \in \mathbb{R}_+$ can be calculated explicitly as

$$\mathrm{Orb}(x, T) = \{(1 + \gamma)^n x : n \geq 0\}.$$

Thus, the orbit tends to 0, x and ∞ for $-1 < \gamma < 0$, $\gamma = 0$ and $\gamma > 0$, respectively.

The orbit of $x \in \mathbb{R}_+$ under the map $T(x) = (1+\gamma)x$ is not generally dense in $\mathbb{R}_+$ Instead, the orbit describes specific behavior depending on the value of γ such that

- For $\gamma > 0$ the orbit grows unbounded as $n \mapsto \infty$, and it is not dense in $\mathbb{R}_+$ because it tends towards infinity.

- For $\gamma = 0$ the orbit remains constant $\mathrm{Orb}(x, T) = \{x\}$, which is a single point, so it is not dense.

- For $-1 < \gamma < 0$ the orbit tends to 0, $n \mapsto \infty$ and again, it is not dense because it converges to a single point.

Example 2. (**The change in behavior of security prices and dividends**).

To consider the behavior of an individual security in the context of a dynamic system, we represent it as a chaotic dynamic system in which the security price and dividends are time-dependent variables.

Let us assume that at time $t-1$ the purchase price of a security is C_{t-1} and at time t the security is sold at the price C_t. During the period t, accrued dividends D_t. Then the rate of return R_t for the period t can be represented as

$$R_t = \frac{C_t + D_t - C_{t-1}}{C_{t-1}}.$$

This value reflects the return over one time period t, which takes into account both the increase in the value of the security itself and the dividends received.

Let the prices C_t and dividends D_t change over time and their behavior depends on previous values. Then we can describe them using a dynamic system, where the state at each step is determined by the state vector $X_t = (C_t, D_t)$, and the evolution of this system over a time period is modeled by the operator T, which goes from state X_{t-1} to X_t

$$X_t = T(X_{t-1}),$$

where the operator T determines the change in price and dividends as a result of the influence of market factors.

To model the chaotic behavior of the system, we can choose the operator T so that the orbit of the system is dense in the space of possible states. This can be done if the operator T is nonlinear and depends on random factors that model market changes, such as economical-financial indicators. For example

$$T(X_{t-1}) = \begin{cases} f(C_{t-1}, D_{t-1}, \varepsilon_t), \\ g(C_{t-1}, D_{t-1}, \varepsilon_t) \end{cases}$$

where f and g are nonlinear functions that take into account the interdependence of prices and dividends, and ε_t is a random variable that adds stochastic market influence.

The addition of random parameters introduces stochasticity into the previously deterministic model. These random parameters represent external market influences such as economic shocks or financial news. This example demonstrates how these systems can be extended to incorporate real-world randomness without losing the fundamental chaotic properties. Specifically, the random operator remains consistent with chaotic principles, as the small perturbations caused by preserve the sensitivity to initial conditions and dense orbit structure, thereby modeling real-world financial data more effectively.

Thus, this system acquires the properties of a chaotic dynamical system with a dense orbit, since the values of C_t and D_t change depending on previous states and random influences. This approach allows modeling complex market dynamics and assessing the average efficiency and risk of a security under uncertainty.

Let the price of a security at time t be denoted by C_t. We can assume that the price dynamics are a function of previous prices, accrued dividends, trading volume, and other market factors. Then we will consider the sequence of values (C_t) as elements of the vector space ℓ_2.

Let T be a shift operator that acts on a sequence of prices (C_t) as follows

$$(TC)_t = \alpha C_{t-1} + \beta C_t + \gamma C_{t+1} + \varepsilon_t,$$

where

- α, β and γ are parameters that adjust the linear combination of prices at different points in time,
- ε_t is a small random variable that takes into account the randomness of market factors.

The inclusion of the random element ε_t adds chaos to the system, and even a linear operator with a shift can generate complex dynamics.

For an operator to be hypercyclic in the space ℓ_2 the coefficients must contribute to the operator's orbits being dense in that space. One known way to ensure hypercyclicity of a shift operator in the space ℓ_2 is to choose coefficients that provide sufficient stretching and shifting of the sequences of values C_t in space, as well as sensitivity to initial conditions.

Consider the operator T for sequences in ℓ_2 of the following form

$$(TC)_t = \alpha C_{t-1} + \beta C + \gamma C_{t+1} + \varepsilon_t,$$

where α, β, γ is the coefficients that we choose ε_t is a small random component.

For hypercyclicity in the space ℓ_2 in particular for the shift operator, the following conditions are important

1. *Zero coefficient at C_t.* It is important that the operator has no constant component (i.e., no fixed value at C_t, as this can limit the dynamics of the operator. Therefore, we set $\beta = 0$. This allows the operator to act as a “shift" by one position in the sequence.
2. *Nonzero values of α and γ.* The coefficients α and γ must be nonzero and have values that do not converge to zero or a stable value. For example, if we choose $\alpha = 1$ and $\gamma = 1$ the operator will expand the orbit of the sequence in both directions.
3. *A small random component ε_t.* Adding random fluctuations ε_t depending on time t provides sensitivity to initial conditions. This guarantees that the sequence can reach different states in the space ℓ_2 are bounded stochastic variables introduced to account for minor unpredictable fluctuations in the financial markets. Practically, this means, that by adding small random fluctuations, the system can explore a wider range of possible states within the space. This enhances the model's ability to capture rare or extreme market events (exogenous shock) that deterministic models might overlook.

Thus, for a hypercyclic operator in the space ℓ_2 we can choose the following values of the coefficients

$$\alpha = 1, \quad \beta = 0, \quad \gamma = 1.$$

Then the operator will have the next form

$$(TC)_t = C_{t-1} + C_{t+1} + \varepsilon_t$$

This configuration with a random perturbation ε_t will allow the operator to generate a dense orbit in the space ℓ_2, and will also provide sensitivity to initial conditions, which is a necessary property for hypercyclicity.

To evaluate the norm of operator T in the space ℓ_2 by the formula

$$\|T\| = \sup_{C\in\ell_2, \|C\|=1} \|TC\|,$$

where $\|C\| = \sqrt{\sum_{t=1}^{\infty} |C_t|^2}$ *is the norm of the sequence* C.

For the operator T defined by the following form

$$(TC)_t = C_{t-1} + C_{t+1} + \varepsilon_t$$

the norm con be calculated by considering the contributions from the terms C_{t-1} and C_{t+1}. The random component ε_t is typically small and bounded, so its contribution to the norm is negligible for estimation purposes.

Then

$$\|TC\|^2 = \sum_{t=1}^{\infty} |(TC)_t|^2 = \sum_{t=1}^{\infty} |C_{t-1} + C_{t+1} + \varepsilon_t|^2.$$

Expanding the square and ignoring higher-order terms of ε_t, we get

$$\|TC\|^2 \approx \sum_{t=1}^{\infty} (|C_{t-1}|^2 + |C_{t+1}|^2).$$

We use the shift properties of sequences in Hilbert space ℓ_2

1) $\sum_{t=1}^{\infty} |C_{t-1}|^2 = \sum_{t=1}^{\infty} |C_t|^2 = \|C\|^2 = 1;$

2) $\sum_{t=1}^{\infty} |C_{t+1}|^2 = \|C\|^2 = 1.$

Thus

$$\|TC\|^2 \approx 2\|C\|^2 = 2,$$
$$\|T\| = \sqrt{2}.$$

We consider the classical Black-Scholes model [11]. This model is used to evaluate the value of financial derivatives, such as options

$$C = S_0 N(d_1) - Ke^{-rt} N(d_2),$$

where C is price of the option, S_0 is current price of the underlying asset, K is strike price of the option, t is time to maturity, r is risk-free interest rate, $N(d_1)$ and $N(d_2)$ are cumulative distribution functions of the standard normal distribution. The parameters d_1 and d_2 are defined as $d_1 = \frac{ln(\frac{S_0}{X})+(r+\frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$, $d_2 = d_1 - \sigma\sqrt{t}$, where σ represents the volatility of the underlying asset.

The Black-Scholes model is primarily used to price financial options and provides a theoretical framework for understanding the relationship between key financial variables. This model is based on the assumption that the price of the underlying asset. follows a geometric Brownian motion. Utilizes stochastic differential equations to describe price dynamics. Designed to price derivatives, specifically European options, rather than modeling the dynamics of the asset price itself. Focuses on determining the fair value of options based on risk-free rates, volatility, and time to maturity. The dynamics model with a shift operator views the price sequence C_t as elements of the vector space ℓ_2, emphasizing time series representation. Assumes that prices are influenced by a linear combination of past, current, and future prices, with parameters α, β, γ determining the weights. Includes a random component ε_t to account for market randomness. Primarily focuses on modeling the asset price's behavior directly rather than pricing derivatives. The Black-Scholes model is the best suited for pricing derivatives like European options or conducting risk analysis in derivative markets. Model with a shift operator is flexible in incorporating market-specific factors. Ideal for studying the behavior of asset prices, identifying trends and forecasting future price movements.

Conclusions In this paper, we have explored the chaotic properties of weighted shift operators on (non-separable) Hilbert spaces. Specifically, we investigated conditions under which these operators exhibit Li-Yorke chaos. Our study examined various structural aspects of the operators that contribute to their chaotic behavior, emphasizing the interplay between the weights and the underlying Hilbert space. Furthermore, we constructed a chaotic dynamical system to model the behavior of security prices. The study bridges the gap between abstract operator theory and applied financial modeling. The chaotic properties of weighted shifts effectively model irregular price behaviors in financial securities. Introducing stochastic parameters aligns the model with real-world data without compromising its chaotic structure. This work provides new insights into the relationship between operator theory and dynamic systems, offering a foundation for future research in both mathematical and applied contexts.

REFERENCES

[1] F. Bayart, E. Matheron, *Dynamics of linear operators*, Cambridge University Press, New York, 2009. https://doi.org/10.1017/CBO9780511581113

[2] T. Bermúdez, N.J. Kalton, *The range of operators on von Neumann algebras*, Proc. Amer. Math. Soc. 2002, **130**, 1447-1455. https://doi.org/10.1090/S0002-9939-01-06292-X.

[3] J. Bes, A. Peris, *Hereditarily hypercyclic operators*, J. Func. Anal. 1999, **167**, 94-112.

[4] G.D. Birkhoff, *Démonstration d'un théorème élémentaire sur les fonctions entières*, C. R. Acad. Sci. Paris 1929, **189**, 473-475.

[5] K.C. Chan and J.H. Shapiro, *The cyclic behavior of translation operators on Hilbert spaces of entire functions,* Indiana Univ. Math. J. 1991, **40** , 1421–1449.

[6] R.L. Devaney, *An introduction to chaotic dynamical systems* Addison-Wesley, Reedwood City, 1989.

[7] K. G. Grosse-Erdmann, A. Peris Manguillot, *Linear chaos*, Springer-Verlag, London, 2011. https://doi.org/10.1007/978-1-4471-2170-1-5

[8] Z. Novosad, Topological transitivity of translation operators in a non-separable Hilbert space. *Carpathian Math. Publ.* **2023**, *15*, 278–285. https://doi.org/10.15330/cmp.15.1.278-285.

[9] Z. Novosad, A. Zagorodnyuk, The Backward Shift and Two Infinite-Dimension Phenomena in Banach Spaces. Symmetry. 2023, **15** P. 1855. https://doi.org/10.3390/sym15101855

[10] N.H. Salas, *Hypercyclic weighted shifts*, Trans. Amer. Math. Soc. 1995, **347**, 993-1004. https://doi.org/10.1090/S0002-9947-1995-1249890-6

[11] T. Worral, *Financial Instruments*, School of Economic and Management Studies. Keele University. Fin-40008. Session 2007/08. https://timworrall.com/fin-40008/index.htm

[12] A. Zagorodnyuk, Z. Novosad, *Topological Transitivity of Shift Similar Operators on Non-separable Hilbert Spaces* Journal of Function Spaces. 2021, Article ID 306342, 7 pages. https://doi.org/10.1155/2021/9306342.

Received 30.11.2024

Новосад З.Г. *Хаотичні динамічні системи операторів зсуву та їх застосування в економіці* // Буковинський матем. журнал — 2024. — Т.12, №2. — С. 37–48.

У статті досліджуються хаотичні властивості операторів зваженого зсуву, які діють на (несепарабельному) гільбертовому просторі, що є одним із важливих об'єктів у теорії динамічних систем. Особливу увагу приділено аналізу умов, за яких такі оператори можуть бути топологічно транзитивними, гіперциклічними і часто гіперциклічними. Крім того, досліджено феномен хаосу Лі-Йорка, який передбачає існування незліченних множин точок із хаотичною поведінкою орбіт. Це дозволяє глибше зрозуміти природу динамічних систем, що характеризуються нерегулярністю і непередбачуваністю.

У статті висвітлюються, як різні властивості операторів зваженого зсуву впливають на їхню динамічну поведінку, розглядаючи взаємодію між вагами оператора та структурою базового простору. Для ілюстрації запропоновано два приклади динамічних систем, які можна використовувати для моделювання поведінки цін на фінансових ринках. Перший приклад базується на простій лінійній моделі, де зміна ціни пропорційна поточному значенню. Побудована орбіта в цьому прикладі в загальному випадку не є щільною. У другому прикладі моделюється більш складна система, яка враховує залежність зміни ціни від попередніх значень, дивідендів та випадкових факторів. У цьому контексті оператор зваженого зсуву відіграє ключову роль, дозволяючи створити гіперциклічну динамічну систему, здатну адекватно відображати хаотичну поведінку цін.

Застосування теорії хаосу до фінансових ринків є особливо актуальним, оскільки це дозволяє враховувати складну динаміку, нелінійність та вплив випадкових факторів на цінові зміни. Використання таких моделей може допомогти інвесторам краще розуміти природу ризиків, знаходити можливості для інвестицій та приймати більш обґрунтовані рішення в умовах невизначеності. Отримані результати мають також важливе значення для широкого спектра наукових досліджень у галузях математики, фізики та економіки, де вивчення хаотичних властивостей систем є центральним для розуміння їхньої поведінки.